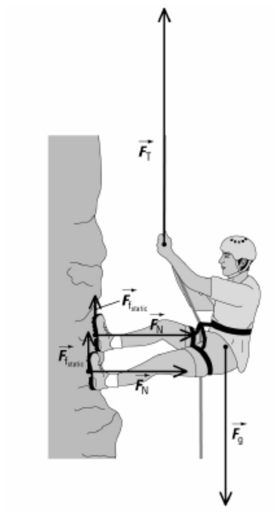
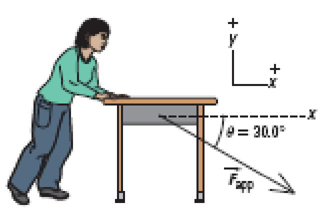
Friction

1) A mountain climber stops during the vertical ascent of a mountain face. Sketch all the forces acting on the climber, and *w h e r e* those forces are acting.



2) The magnitude of the applied force on a desk is 165 N and 30.0° below the horizontal. If the desk remains stationary, calculate the force of static friction acting on the desk. [143 N [180°]]



x direction

$$\vec{F}_{net_x} = \vec{F}_{app_x} + \vec{F}_{f_{static}}$$
$$F_{net_x} = F_{app_x} + F_{f_{static}}$$
$$0 = F_{app_x} + F_{f_{static}}$$

y direction

$$\vec{F}_{net_y} = \vec{F}_N + \vec{F}_{app_y} + \vec{F}_g$$
$$F_{net_y} = 0$$

Calculations in the *y* direction are not required in this problem.

$$\begin{aligned} F_{f_{static}} &= -F_{app_x} \\ &= -(165\ \text{N})(\cos\theta) \\ &= -(165\ \text{N})(\cos 30.0°) \\ &= -143\ \text{N} \end{aligned}$$

$\vec{F}_{f_{static}}$ prevents the desk from sliding in the *x* direction. The negative value for $F_{f_{static}}$ indicates that the direction of $\vec{F}_{f_{static}}$ is along the negative *x*-axis or [180°].

$$\vec{F}_{f_{static}} = 143\ \text{N [180°]}$$

3) A force of 31 N [forward] is needed to start an 8.0-kg steel slider moving along a horizontal steel rail. What is the coefficient of static friction? [0.40]

$\vec{F}_{app} = 31$ N [forward]
$\vec{g} = 9.81\ \text{m/s}^2$ [down]
$m = 8.0$ kg

Required

coefficient of static friction (μ_s)

Since the slider is not accelerating, $\vec{F}_{net} = 0$ N in both the horizontal and vertical directions. Write equations to find the net force on the slider in both directions.

horizontal direction

$$\begin{aligned}
\vec{F}_{net_h} &= \vec{F}_{app} + \vec{F}_{f_{static}} \\
F_{net_h} &= F_{app} + F_{f_{static}} \\
0 &= 31\text{ N} + (-\mu_s F_N) \\
&= 31\text{ N} - \mu_s F_N \\
\mu_s F_N &= 31\text{ N}
\end{aligned}$$

vertical direction

$$\begin{aligned}
\vec{F}_{net_v} &= \vec{F}_N + \vec{F}_g \\
F_{net_v} &= F_N + F_g \\
0 &= F_N + (-mg) \\
&= F_N - mg \\
F_N &= mg
\end{aligned}$$

Substitute $F_N = mg$ into the last equation for the horizontal direction.

$$\begin{aligned}
\mu_s mg &= 31\text{ N} \\
\mu_s &= \frac{31\text{ N}}{mg}
\end{aligned}$$

$$\begin{aligned}
\mu_s &= \frac{31\text{ N}}{(8.0\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} \\
&= \frac{31\ \cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{\left(8.0\ \cancel{\text{kg}}\right)\left(9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)} \\
&= 0.40
\end{aligned}$$

4) A biker and his motorcycle have a weight of 2350 N [down]. Calculate the force of kinetic friction for the rubber tires and dry concrete if the motorcycle skids. [2.0×10^3 N [backward]]

$\vec{F}_g = 2350$ N [down] $\qquad \vec{g} = 9.81\text{ m/s}^2$ [down]

$\mu_k = 0.7$ from Table 3.4 (rubber tires on dry concrete)

Required

force of kinetic friction ($\vec{F}_{f_{kinetic}}$)

Since the system is skidding, $\vec{F}_{net} \neq 0$ N in the horizontal direction, but $\vec{F}_{net} = 0$ N in the vertical direction.

Calculate $F_{f_{kinetic}}$.

$$\begin{aligned}
F_{f_{kinetic}} &= -\mu_k F_N \\
&= -\mu_k mg \\
&= -(0.7)(2350\text{ N}) \\
&= -2 \times 10^3\text{ N} \\
\vec{F}_{f_{kinetic}} &= 2 \times 10^3\text{ N [backward]}
\end{aligned}$$

5) A force of 15 N [S] moves a case of soft drinks weighing 40 N [down] across a level counter at constant velocity. Calculate the coefficient of kinetic friction for the case on the counter. [0.38]

$\vec{F}_{app} = 15$ N [S]
$\vec{F}_g = 40$ N [down]
$\vec{a} = 0$ m/s^2

Required
coefficient of kinetic friction (μ_k)

Since the case is moving at constant speed, $\vec{F}_{net} = 0$ N in both the horizontal and vertical directions.
Write equations to find the net force on the case in both directions.

horizontal direction

$$\vec{F}_{net_h} = \vec{F}_{app} + \vec{F}_{f_{kinetic}}$$
$$F_{net_h} = F_{app} + F_{f_{kinetic}}$$
$$0 = 15\text{ N} + (-\mu_k F_N)$$
$$= 15\text{ N} - \mu_k F_N$$
$$\mu_k F_N = 15\text{ N}$$

vertical direction

$$\vec{F}_{net_v} = \vec{F}_N + \vec{F}_g$$
$$F_{net_v} = F_N + F_g$$
$$0 = F_N + (-40\text{ N})$$
$$= F_N - 40\text{ N}$$
$$F_N = 40\text{ N}$$

Substitute $F_N = 40$ N into the last equation for the horizontal direction.

$$\mu_k(40\text{ N}) = 15\text{ N}$$
$$\mu_k = \frac{15\ \cancel{\text{N}}}{40\ \cancel{\text{N}}}$$
$$= 0.38$$

6) An applied force of 450 N [forward] is needed to drag a 1000-kg crate at constant speed across a horizontal, rough floor. Calculate the coefficient of kinetic friction for the crate on the floor. [4.59 x 10^{-2}]

$$\begin{aligned}\vec{F}_{app} &= 450\ \text{N [forward]}\\ \vec{g} &= 9.81\ \text{m/s}^2\ \text{[down]}\\ m &= 1000\ \text{kg}\\ \vec{a} &= 0\ \text{m/s}^2\end{aligned}$$

Required

coefficient of kinetic friction (μ_k)

Since the crate is moving at constant speed, $\vec{F}_{net} = 0$ N in both the horizontal and vertical directions.
Write equations to find the net force on the crate in both directions.

horizontal direction

$$\begin{aligned}\vec{F}_{net_h} &= \vec{F}_{app} + \vec{F}_{f_{kinetic}}\\ F_{net_h} &= F_{app} + F_{f_{kinetic}}\\ 0 &= 450\ \text{N} + (-\mu_k F_N)\\ &= 450\ \text{N} - \mu_k F_N\\ \mu_k F_N &= 450\ \text{N}\end{aligned}$$

vertical direction

$$\begin{aligned}\vec{F}_{net_v} &= \vec{F}_N + \vec{F}_g\\ F_{net_v} &= F_N + F_g\\ 0 &= F_N + (-mg)\\ &= F_N - mg\\ F_N &= mg\end{aligned}$$

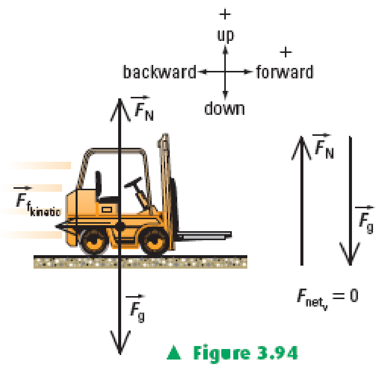
Substitute $F_N = mg$ into the last equation for the horizontal direction.

$$\begin{aligned}\mu_k mg &= 450\ \text{N}\\ \mu_k &= \frac{450\ \text{N}}{mg}\\ &= \frac{450\ \text{N}}{(1000\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\\ &= \frac{450\ \cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{(1000\ \cancel{\text{kg}})\left(9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)}\\ &= 4.59\times 10^{-2}\end{aligned}$$

7) A 1640-kg forklift with rubber tires is skidding on wet concrete with all four wheels locked. Calculate the acceleration of the truck. [5 m/s^2 [backwards]]

$m = 1640$ kg $\qquad \vec{g} = 9.81$ m/s² [down]

$\mu_k = 0.5$ from Table 3.4 (rubber on wet concrete)



▲ **Figure 3.94**

horizontal direction

$$\begin{aligned}
\vec{F}_{net_h} &= \vec{F}_{f_{kinetic}} \\
F_{net_h} &= F_{f_{kinetic}} \\
ma &= F_{f_{kinetic}} \\
&= -\mu_k F_N
\end{aligned}$$

vertical direction

$$\begin{aligned}
\vec{F}_{net_v} &= \vec{F}_N + \vec{F}_g \\
F_{net_v} &= F_N + F_g \\
0 &= F_N + (-mg) \\
&= F_N - mg \\
F_N &= mg
\end{aligned}$$

Substitute $F_N = mg$ into the equation for $F_{f_{kinetic}}$.

$$\begin{aligned}
\cancel{m}a &= -\mu_k \cancel{m}g \\
a &= -\mu_k g \\
&= -(0.5)\left(9.81 \frac{\text{m}}{\text{s}^2}\right) \\
&= -5 \text{ m/s}^2
\end{aligned}$$

The negative value for a indicates that the direction of $\vec{a}$ is backward.

$\vec{a} = 5$ m/s² [backward]

8) A pair of skis weigh 15 N [down]. Calculate the difference in the maximum force of static friction for the skis on a wet and dry snowy, horizontal surface. [2 N [backward]]

$\vec{F}_g$ = 15 N [down]

μ_s = 0.06 from Table 3.4 (waxed hickory skis on dry snow)

μ_s = 0.20 from Table 3.4 (waxed hickory skis on wet snow)

Required

difference in the maximum force of static friction on wet and dry snow ($\Delta \vec{F}_{f_{static}}$)

Since the skis are not accelerating, $\vec{F}_{net}$ = 0 N both in the horizontal and vertical directions.

Calculate $F_{f_{static}}$ on each surface.

$$\begin{aligned} F_{f_{static}} &= -\mu_s F_N \\ &= -\mu_s mg \end{aligned}$$

Dry Snow

$$\begin{aligned} F_{f_{static}} &= -\mu_s mg \\ &= -(0.06)(15\ \text{N}) \\ &= -0.90\ \text{N} \\ \vec{F}_{f_{static}} &= 0.90\ \text{N [backward]} \end{aligned}$$

Wet Snow

$$\begin{aligned} F_{f_{static}} &= -\mu_s mg \\ &= -(0.20)(15\ \text{N}) \\ &= -3.0\ \text{N} \\ \vec{F}_{f_{static}} &= 3.0\ \text{N [backward]} \end{aligned}$$

Calculate the difference between the two values of $F_{f_{static}}$.

$$\begin{aligned} \Delta F_{f_{static}} &= 3.0\ \text{N} - 0.90\ \text{N} \\ &= 2\ \text{N} \\ \Delta \vec{F}_{f_{static}} &= 2\ \text{N [backward]} \end{aligned}$$

9) A sled with waxed hickory runners rests on a horizontal, dry snowy surface. Calculate the mass of the sled if the maximum force that can be applied to the sled before it starts moving is 46 N [forward]. [8 x 10^1 kg]

Given

$\vec{F}_{\text{app}} = 46 \text{ N [forward]}$ $\quad\quad \vec{g} = 9.81 \text{ m/s}^2 \text{ [down]}$

$\mu_s = 0.06$ from Table 3.4 (waxed hickory skis on dry snow)

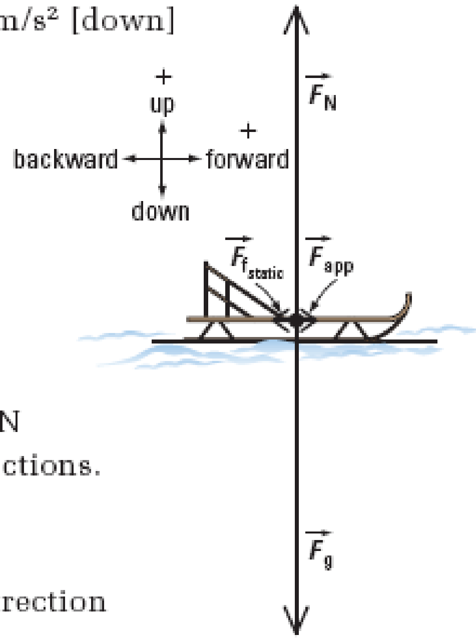
Required

mass of sled (m)

Analysis and Solution

Draw a free-body diagram for the sled (Figure 3.89).

Since the sled is not accelerating, $\vec{F}_{\text{net}} = 0 \text{ N}$ in both the horizontal and vertical directions. Write equations to find the net force on the sled in both directions.



▲ **Figure 3.89**

horizontal direction

$$\begin{aligned}
\vec{F}_{\text{net}_\text{h}} &= \vec{F}_{\text{app}} + \vec{F}_{\text{f}_\text{static}} \\
F_{\text{net}_\text{h}} &= F_{\text{app}} + F_{\text{f}_\text{static}} \\
0 &= F_{\text{app}} + F_{\text{f}_\text{static}} \\
&= F_{\text{app}} + (-\mu_s F_\text{N}) \\
&= F_{\text{app}} - \mu_s F_\text{N} \\
F_{\text{app}} &= \mu_s F_\text{N}
\end{aligned}$$

vertical direction

$$\begin{aligned}
\vec{F}_{\text{net}_\text{v}} &= \vec{F}_\text{N} + \vec{F}_\text{g} \\
F_{\text{net}_\text{v}} &= F_\text{N} + F_\text{g} \\
0 &= F_\text{N} + (-mg) \\
&= F_\text{N} - mg \\
F_\text{N} &= mg
\end{aligned}$$

Substitute $F_\text{N} = mg$ into the equation for F_{app}.

$$\begin{aligned}
F_{\text{app}} &= \mu_s mg \\
m &= \frac{F_{\text{app}}}{\mu_s g} \\
&= \frac{46 \text{ N}}{(0.06)\left(9.81 \dfrac{\text{m}}{\text{s}^2}\right)} \\
&= \frac{46 \text{ kg}\cdot\dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{(0.06)\left(9.81 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)} \\
&= 8 \times 10^1 \text{ kg}
\end{aligned}$$

10) Suppose the sled above is resting on a horizontal, wet snowy surface. Would the sled move if the applied force is 125 N? Explain. [No, $F_{static} > F_a$]

$\vec{F}_{\text{app}} = 125$ N [forward]

$\vec{g} = 9.81$ m/s^2 [down]

$\mu_s = 0.20$ from Table 3.4 (waxed hickory skis on wet snow)

$m = 78$ kg from Example 3.17

Since the sled is not accelerating in the vertical direction, $F_{\text{net}_\text{h}} = 0$ N.

Write equations to find the net force on the sled in both directions.

horizontal direction

$$
\begin{aligned}
\vec{F}_{\text{net}_\text{h}} &= \vec{F}_{\text{app}} + \vec{F}_{\text{f}_\text{static}} \\
F_{\text{net}_\text{h}} &= F_{\text{app}} + F_{\text{f}_\text{static}} \\
&= F_{\text{app}} + F_{\text{f}_\text{static}} \\
&= F_{\text{app}} + (-\mu_s F_\text{N}) \\
&= F_{\text{app}} - \mu_s F_\text{N}
\end{aligned}
$$

vertical direction

$$
\begin{aligned}
\vec{F}_{\text{net}_\text{v}} &= \vec{F}_\text{N} + \vec{F}_\text{g} \\
F_{\text{net}_\text{v}} &= F_\text{N} + F_\text{g} \\
0 &= F_\text{N} + (-mg) \\
&= F_\text{N} - mg \\
F_\text{N} &= mg
\end{aligned}
$$

Substitute $F_\text{N} = mg$ into equation for F_{net_h}.

$$
\begin{aligned}
F_{\text{net}_\text{h}} &= F_{\text{app}} - \mu_s mg \\
&= 125 \text{ N} - (0.20)(78 \text{ kg})(9.81 \text{ m/s}^2) \\
&= -28 \text{ N}
\end{aligned}
$$

Since $\vec{F}_{\text{net}_\text{h}}$ is negative, $F_{\text{f}_\text{static}} > F_{\text{app}}$.

Paraphrase

The sled will not move on a wet snowy surface if an applied force of 125 N acts on it.

11) An applied force of 24 N [forward] causes a steel block to start moving across a horizontal, greased steel surface. Calculate the mass of the block. [16 kg]

$\vec{F}_{app}$ = 24 N [forward]
$\vec{g}$ = 9.81 m/s^2 [down]
μ_s = 0.15 from Table 3.4 (steel on greased steel)

Since the block is not accelerating, $\vec{F}_{net} = 0$ N in both the horizontal and vertical directions. Write equations to find the net force on the block in both directions.

horizontal direction

$$\begin{aligned}\vec{F}_{net_h} &= \vec{F}_{app} + \vec{F}_{f_{static}}\\ F_{net_h} &= F_{app} + F_{f_{static}}\\ 0 &= F_{app} + F_{f_{static}}\\ &= F_{app} + (-\mu_s F_N)\\ &= F_{app} - \mu_s F_N\\ F_{app} &= \mu_s F_N\end{aligned}$$

vertical direction

$$\begin{aligned}\vec{F}_{net_v} &= \vec{F}_N + \vec{F}_g\\ F_{net_v} &= F_N + F_g\\ 0 &= F_N + (-mg)\\ &= F_N - mg\\ F_N &= mg\end{aligned}$$

Substitute $F_N = mg$ into equation for F_{app}.

$$\begin{aligned}F_{app} &= \mu_s mg\\ m &= \frac{F_{app}}{\mu_s g}\\ &= \frac{24\ \text{N}}{(0.15)\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}\\ &= \frac{24\ \text{kg}\cdot\cancel{\frac{\text{m}}{\text{s}^2}}}{(0.15)\left(9.81\ \cancel{\frac{\text{m}}{\text{s}^2}}\right)}\\ &= 16\ \text{kg}\end{aligned}$$

12) A tractor and tow truck have rubber tires on wet concrete. The tow truck drags the tractor at constant velocity while its brakes are locked. If the tow truck exerts a horizontal force of 1.0 X 10^4 N on the tractor, determine the mass of the tractor. [2 x 10^3 kg]

$\vec{a} = 0\ \text{m/s}^2$

$\vec{F}_{\text{app}} = 1.0 \times 10^4\ \text{N [forward]}$

$\vec{g} = 9.81\ \text{m/s}^2\ \text{[down]}$

$\mu_k = 0.5$ from Table 3.4 (rubber tires on wet concrete)

Required

mass of tractor (m)

Since the tractor is not accelerating in the vertical direction, $F_{\text{net}_h} = 0$ N.

Write equations to find the net force on the tractor in both directions.

horizontal direction

$$\begin{aligned}
\vec{F}_{\text{net}_h} &= \vec{F}_{\text{app}} + \vec{F}_{\text{f}_{\text{kinetic}}} \\
F_{\text{net}_h} &= F_{\text{app}} + F_{\text{f}_{\text{kinetic}}} \\
0 &= F_{\text{app}} + F_{\text{f}_{\text{kinetic}}} \\
&= F_{\text{app}} + (-\mu_k F_N) \\
&= F_{\text{app}} - \mu_k F_N \\
\mu_k F_N &= F_{\text{app}}
\end{aligned}$$

vertical direction

$$\begin{aligned}
\vec{F}_{\text{net}_v} &= \vec{F}_N + \vec{F}_g \\
F_{\text{net}_v} &= F_N + F_g \\
0 &= F_N + (-mg) \\
&= F_N - mg \\
F_N &= mg
\end{aligned}$$

Substitute $F_N = mg$ into equation for F_{app}.

$$\mu_k mg = F_{\text{app}}$$

$$\begin{aligned}
m &= \frac{F_{\text{app}}}{\mu_k g} \\
&= \frac{1.0 \times 10^4\ \text{N}}{\mu_k g} \\
&= \frac{1.0 \times 10^4\ \text{kg} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{(0.5)\left(9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)} \\
&= 2 \times 10^3\ \text{kg}
\end{aligned}$$

13) A warehouse employee applies a force of 120 N [12.0°] to accelerate a 35-kg wooden crate from rest across a wooden floor. The coefficient of kinetic friction for the crate on the floor is 0.30. How much time elapses from the time the employee starts to move the crate until it is moving at 1.2 m/s [0°]. [1.9 s]

Given

$\vec{F}_{app}$ = 120 N [12.0°]
m = 35 kg
μ_k = 0.30
$\vec{g}$ = 9.81 m/s^2 [down]
$\vec{v}_f$ = 1.2 m/s [0°]

Required

elapsed time (Δt)

Resolve $\vec{F}_{app}$ into x and y components.

Vector	x component	y component
$\vec{F}_{app}$	(120 N)(cos 12.0°)	(120 N)(sin 12.0°)

Since the crate is accelerating in the x direction, $F_{net_x} \neq 0$ N but $F_{net_y} = 0$ N.

Write equations to find the net force on the crate in the x and y directions.

x direction

$$\begin{aligned} \vec{F}_{net_x} &= \vec{F}_{app_x} + \vec{F}_{f_{kinetic}} \\ F_{net_x} &= F_{app_x} + F_{f_{kinetic}} \\ ma &= (120\text{ N})(\cos 12.0°) + (-\mu_k F_N) \\ &= (120\text{ N})(\cos 12.0°) - \mu_k F_N \end{aligned}$$

y direction

$$\begin{aligned} \vec{F}_{net_y} &= \vec{F}_N + \vec{F}_{app_y} + \vec{F}_g \\ F_{net_y} &= F_N + F_{app_y} + F_g \\ 0 &= F_N + (120\text{ N})(\sin 12.0°) + (-mg) \\ F_N &= mg - (120\text{ N})(\sin 12.0°) \\ &= (35\text{ kg})(9.81\text{ m/s}^2) \\ &\quad - (120\text{ N})(\sin 12.0°) \\ &= 318\text{ N} \end{aligned}$$

Substitute F_N = 318 N into the last equation for the x direction.

$$\begin{aligned} ma &= (120\text{ N})(\cos 12.0°) - (0.30)(318\text{ N}) \\ &= 21.9\text{ N} \\ a &= \frac{21.9\text{ N}}{35\text{ kg}} \\ &= 0.625\text{ m/s}^2 \end{aligned}$$

The crate is accelerating along the 0° direction.

$$\vec{a} = 0.625\text{ m/s}^2\ [0°]$$

Calculate the elapsed time.

$$\begin{aligned} \vec{a} &= \frac{\Delta \vec{v}}{\Delta t} \\ &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \\ 0.625\text{ m/s}^2 &= \frac{1.2\text{ m/s} - 0}{\Delta t} \\ \Delta t &= \frac{1.2\ \frac{\cancel{m}}{\cancel{s}}}{0.625\ \frac{\cancel{m}}{s^{\cancel{2}}}} \\ &= 1.9\text{ s} \end{aligned}$$

Paraphrase

The elapsed time will be 1.9 s.

14) A loaded dogsled has a mass of 400 kg and is being pulled across a horizontal, packed snow surface at a velocity of 4.0 m/s [N]. Suddenly, the harness separates from the sled. If the coefficient of kinetic friction for the sled on the snow is 0.0500, how far will the sled coast before stopping? [16 m]

$m = 400$ kg
$\vec{g} = 9.81$ m/s^2 [down]
$\vec{v}_i = 4.0$ m/s [N]
$\mu_k = 0.0500$

Required

coasting distance of sled (d)

Analysis and Solution

Draw a free-body diagram for the sled.

Since the sled is coasting, $\vec{F}_{net} \neq 0$ N in the horizontal direction, but $\vec{F}_{net} = 0$ N in the vertical direction.

Write equations to find the net force on the sled in both directions.

horizontal direction

$$\vec{F}_{net_h} = \vec{F}_{f_{kinetic}}$$
$$F_{net_h} = F_{f_{kinetic}}$$
$$ma = -\mu_k F_N$$

vertical direction

$$\vec{F}_{net_v} = \vec{F}_N + \vec{F}_g$$
$$F_{net_v} = F_N + F_g$$
$$0 = F_N + (-mg)$$
$$= F_N - mg$$
$$F_N = mg$$

Substitute $F_N = mg$ into the last equation for the horizontal direction.

$$\cancel{m}\,a = -\mu_k \cancel{m}\, g$$
$$a = -\mu_k g$$
$$= -(0.0500)(9.81 \text{ m/s}^2)$$
$$= -0.49 \text{ m/s}^2$$
$$\vec{a} = 0.49 \text{ m/s}^2 \text{ [S]}$$

Since the sled coasts to a stop, calculate the coasting distance of the sled.

$$(v_f)^2 = (v_i)^2 + 2ad$$
$$0 = (v_i)^2 + 2ad$$
$$2ad = -(v_i)^2$$
$$d = \frac{-(v_i)^2}{2a}$$
$$= \frac{-(4.0 \text{ m/s})^2}{2(-0.49 \text{ m/s}^2)}$$
$$= 16 \text{ m}$$

Paraphrase

The sled will coast for 16 m before it stops.